



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 19

Immunization & Bond Dynamics

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ILLUSTRATION

- We consider a 12% annual coupon bond of face & redemption value 1,000 with 25 years to maturity.
- The $t=0$ interest rates (YTM) are 16.65%.
- At this YTM, the CMP is 726.66 and duration is 7 years.

- It means that for $y=16.65\%$ and holding period $H=D=7$ years, the first derivative of $TCF(y,H)$ with respect to y will be zero.
- This means that the TCF value that we get i.e. $TCF(16.65\%, 7 \text{ years})=2135.63$ is independent of “SMALL” changes in interest rates around 16.65%, since the derivative is with respect to interest rates.

TOTAL CASH FLOWS VS REINVESTMENT RATES IF THE BOND IS HELD FOR ITS DURATION.

- I have fixed the holding period at 7 years (equal to duration) and worked out the Total Cash Flows at 7 years for different reinvestment rates: The results are as follows:

R (%)	TCF	R(%)	TCF
11	2251	16	2137
12	2210	16.65	2135.63
13	2180	17	2135.94
14	2158	18	2141
15	2144	19	2150



- There are two things worth noting here:
- For interest rates close to the YTM e.g. 16% and 17% the variation in cash flows is very small if the bond is held for its duration.
- We have the minimum total cash flows for the holding period equal to the duration.
- This is the way it should be because duration constitutes the minimum risk holding period in the sense that the curve is flattest at this point and variation in TCF is minimum around this point.



- **Let us investigate further:**
- **Let us say that A has bought the bond at $t=0$ at a ytm of 16.65% which are the current market rates.**
- **A will get a yield of 16.65% if interest rates do not change.**

- If interest rates do not change, the $t=0$ investor (A) earns a return of 16.65% if:
- all coupons are reinvested at ytm of 16.65% and
- (i) either he holds the bond to maturity; or
- (ii) he liquidates the bond at an earlier date at a price with a ytm of 16.65%.
- In other words, irrespective of holding period provided all coupons and sale price are calculated at 16.65%.

- What if interest rates change? Obviously, P_0 is fixed being the price at which A have bought the bond.
- If A holds the bond for $t=D=7$ years, his TCF at $t=7$ years is immunized against changes in interest rates.
- Thus, any small change in interest rates will not change the TCF at $t=7$ years i.e. duration.
- This means that cash flows at $t=0$ and $t=7$ are both fixed and independent of small changes in y .
- Thus, the return over 7 years is independent of small changes in y if A holds the bond for 7 years..

BOND DYNAMICS IF INTEREST RATE CHANGES?

- Let the interest rate change to 17% on day $t=1$.
- The price of the bond will change from 726.96 ($t=1$ day price at 16.65%) to 712 ($t=1$ day price at 17%).
- If B buys the bond at $t=1$ day at 712, he will get the YTM return (17%) if all future coupons are reinvested at ytm of 17% and
 - (i) either he holds the bond to maturity; or
 - (ii) he liquidates the bond at an earlier date at a price with a ytm of 17%.

- If A liquidates his investment on day $t=1$, he loses $(712-726.66)/726.66 = (-)2.02\%$ in one day on his investment.
- **THIS IS BECAUSE HE DID NOT HOLD ON TO THE INVESTMENT TILL 7 YEARS (DURATION)**
- If A had held the investment for 7 years, his cash flows (after interest rates changed to 17%) would have been 2135.92 with a return of almost exactly same as his original anticipated return of 16.65%.

- **If A's holding period was 7 years,** then due to the increase in rate from 16.65% to 17%,
- A would have reinvested coupons at this higher rate and hence, get higher interest on reinvested coupons
- The capital loss suffered by A would have reduced because the no of discounting periods at the higher rate of 17% would have been less.
- **For holding period of 7 years, these effects annul each other exactly.**

- Thus, notwithstanding the small change in interest rates, A's return will remain unchanged if he holds the bond for 7 years.
- However, the price of the bond at $t=1$ day will readjust so that over the remaining period i.e. $t=1$ day onwards, the bond provides a YTM of 17%, if interest rates do not change again.

- In other words, if another person B buys the bond after the change in interest rate from 16.65% to 17% i.e. after $t=1$ day, he will get a return of 17% and not 16.65%, if rates do not change again.
- Thus, if A who bought the bond at $t=0$ when interest rate was 16.65% sells it to B after the interest rates changed to 17% i.e. end of $t=1$ day, he will not get the yield of 16.65% but a lower yield.
- **But A did not hold the bond to its duration.**

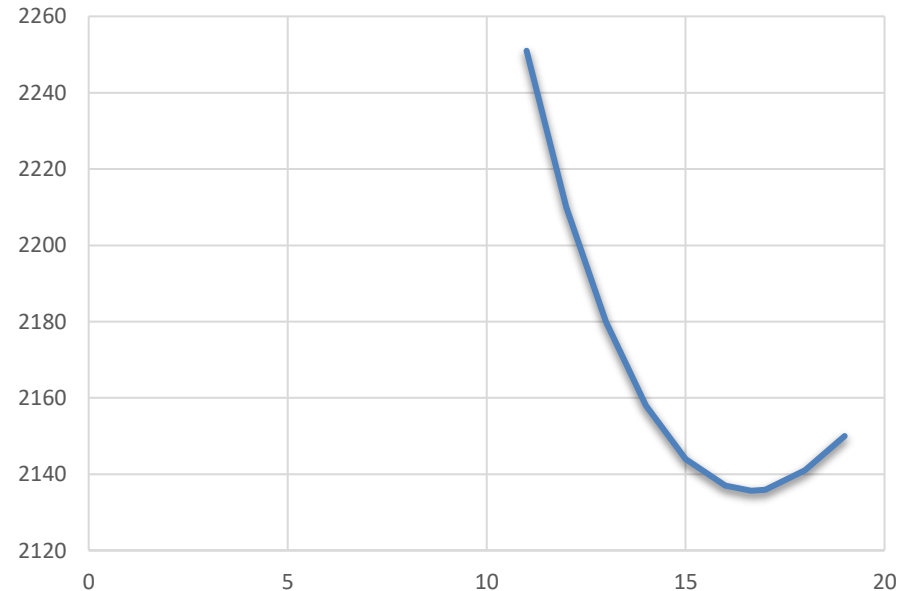
WHAT ABOUT B?

- B takes the bond at 17% after rates have changed on $t=1$ day.
- He will get the YTM of 17% if interest rates do not change.
- However, if interest rates change, he will not be immunized if he holds the bond for 7 years.
- **THIS IS BECAUSE THE DURATION AT 17% (6.90 YEARS) IS NOT THE SAME AS AT 16.65% (7 YEARS).**
- **Hence, he needs to hold the bond for 6.90 years for immunity.**

CAVEATS

- If there is a large change in interest rates, then this independence will not hold since the flatness of the curve extends only for a small region around 16.65%.

TCF VS INTEREST RATE



- Because duration D is y dependent, if we use a different interest rate we shall get a different duration.
- Thus, if we use a different interest rate for calculating the TCF but calculate the TCF at 7 years, we shall NOT be calculating the TCF at its duration and hence, the immunity will not hold.

- This is seen from the example. If we use interest rate of 11% and calculate TCF for 7 years, we get 2251. However, if the interest rate changes to 12% the TCF changes to 2210, a fall of 41. Hence, the immunity does not hold.
- **THIS IS BECAUSE THE DURATION AT 11% IS NOT 7 YEARS. IT IS 9.25 YEARS.**

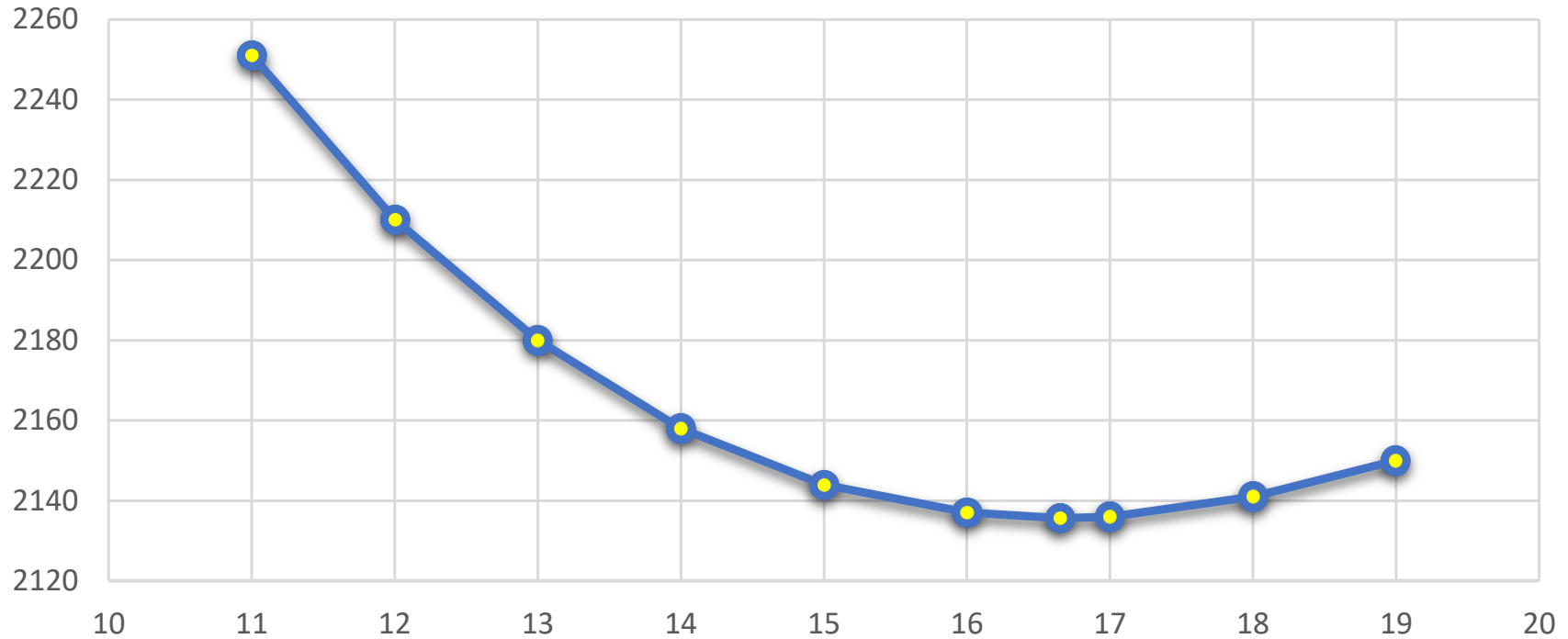
TCF IS A MINIMA AT DURATION

- Because the second derivative of $TCF(16.65\%, 7)$ with respect to y is positive, it follows that the TCF is a minimum at this $t=7$ years for $y=16.65\%$.
- In other words, the TCF will increase on either side of 16.65% for a holding period of 7 years i.e. if the interest rates change but the holding period is 7 years.

- The important thing is that the variation in TCF is minimal around its duration corresponding to a given YTM.
- TCF may increase at rates other than the ytm of 16.65% for 7 year holding period. But then the sensitivity of TCF to interest rate change also increases.

- This is seen below:
- Suppose the ytm at $t=0$ is 11% and you hold the bond for 7 years. Your TCF is 2251. if the interest rate is 12%, the TCF changes to 2210.
- Although the TCF is higher than the TCF at 16.65%, the change in TCF is far more than that around 16.65%. This is commensurate with the maxim of high expected return for high risk and vice versa.

TCF VS INTEREST RATE



DURATION OF ZEROS

- The duration of a zero coupon bond is equal to its maturity.
- Hence, longer term zeros are more price sensitive than short term zeros.
- This makes intuitive sense, since a change in yield for a long term bond affects cash flows over a larger number of discounting periods.

